THE ENGINEERING ECONOMIST

SPRING 1958

VOLUME 3 NUMBER 4

PUBLISHED BY THE ENGINEERING ECONOMY DIVISION
of the
AMERICAN SOCIETY FOR ENGINEERING EDUCATION

THE ENGINEERING ECONOMIST

VOLUME 3
NUMBER 4

SPRING
1958

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CONTENTS

ARTICLES

False and Multiple Solutions by the Discounted Cash Flow Method for Determining Interest Rate of Return
James B. Weaver 1

A Simplified Formula for Break-Even Points
Benjamin Caplan 55

REVIEW ARTICLES

Modernizing the Break-Even Analysis
Ernest H. Weinwurm 32

Ray I. Reul, "Profitability Index for Investments"
Ralph O. Swalm 40

REVIEW

A Sampling of the Recent Literature of Engineering Economics in the Tool Production Engineering Area
Lawrence E. Doyle 47

INDEX TO VOLUME 3: Subject Matter 59
Authors 61

PROGRAM: Engineering Economy Division - ASEE Annual Meeting
Berkeley, California June 1958
See following page.

- * -

The Engineering Economist
A Quarterly Journal

Stevens Institute of Technology
Box L, Hoboken, New Jersey

Subscription $3 a year in the United States and Canada. $4 a year elsewhere. Payable in advance.

PROGRAM
Engineering Economy Division
SIXTY-SIXTH ANNUAL MEETING
American Society for Engineering Education
June 1958
Berkeley, California

Monday, June 16th

2:00 p.m. Conference

Presiding: Professor W. D. McIlvaine, University of Michigan, Assistant to the Dean, College of Engineering
Theme: Capital Budgeting
1st Paper: Capital Budgeting
Speaker: Professor Gordon Shillinglaw, MIT, School of Industrial Management
2nd Paper: Some Practical Aspects of Applying Capital Budgeting Techniques
Speaker: R. W. Johnson, Assistant Comptroller, Standard Oil Company of California, San Francisco, California

Wednesday, June 18th

12:00 Noon Joint Luncheon

Presiding: Professor A. R. Gruehr, Polytechnic Institute of Brooklyn
Paper: Economic Consequences of Unreliability
Speaker: Robert Lusser, Reliability Coordinator, U.S. Army Missile Command, Redstone Arsenal, Alabama

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(Program continued on next page.)

Wednesday, June 18th

(Contd.)

6:00 p.m. Dinner Meeting - Business Meeting - Open

Presiding: Professor A. R. Gruehr, Polytechnic Institute of Brooklyn
Theme: Highway Cost Allocation
Paper: Transportation Engineering and Transportation Economics - One Field or Two
Speaker: D. Gordon Tyndall, University of California, Berkeley, California, Lecturer in Business Administration

BUSINESS MEETING

Thursday, June 19th

2:00 p.m. Conference

Presiding: Professor E. L. Grant, Acting Executive Head, Department of Industrial Engineering, Stanford University
Theme: Sunk Costs
1st Paper: Significant Ideas on the Concept of Sunk Cost
Speaker: Professor Arthur Lesser, Jr., Head, Department of Industrial Engineering, Stevens Institute of Technology
2nd Paper: Implication of Sunk Costs for Managerial Decisions
Speaker: Lawrance F. Bell, Belcraft Manufacturing Company, Palo Alto, California

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FALSE AND MULTIPLE SOLUTIONS BY THE DISCOUNTED CASH FLOW METHOD FOR DETERMINING INTEREST RATE OF RETURN

By

James B. Weaver
Atlas Powder Company

The technique of determining the earning interest rate of return on investment by discounted cash flow analysis is well established in engineering economy and has been described many times in the literature (References 1-5, 7-10, 12, 13, 15, 16, 19, 21-23). It is the purpose of this paper to discuss problems encountered in certain cases by use of this mathematical technique, and to recommend revisions in the technique -- the mathematical model of investment -- which will provide a true interest rate of return in some of these problem cases.

The major problem discussed herein is the occurrence of false and multiple solutions to a rate of return problem, when solved by established methods. Before the problem is discussed, certain definitions and relationships must be presented.

The author is no longer sure (see Reference 22) that interest rate of return is an adequate model for all investment situations; it is clearly stated herein that acceleration problems cannot be analyzed by this technique. However, other analytical techniques such as incremental present worth (References 6 and 23) or payout time including interest (Reference 14) also have disadvantages, particularly that of choosing and/or calculating a "cost of capital" (References 17 and 18) or minimum acceptable rate of return. Rather than summarily abandon the interest rate of return technique, we have undertaken to analyze some of its weaknesses.

If enough thoughts can be brought to bear on all these problems, a single satisfactory technique may eventually be developed.

TERMINOLOGY: INVESTMENT AND CASH FLOW RELATIONSHIPS

For details of the concept of interest rate of return and for the explanation of the usual method of solution by discounted cash flow techniques, described in terminology used below, see Reference 22. These brief notes are not intended to completely instruct those unfamiliar with the method, but only to review certain relationships for those fairly familiar with the subject and to provide familiarity with the author's terminology.

The basic concept of interest rate of return is the application of compound interest rates to all cash flows involved in an undertaking, by trial and error, to determine the average rate of interest at which the company's investment is repaid by proceeds of the project. Note, however, that the calculated rate of interest is not earned on the initial investment throughout the project life. All proceeds from the project, beyond that required for interest, are credited by the method of solution toward repayment of initial investment. Interest for subsequent years is earned only on residual investment.

Chart of Cumulative Cash Position (CCP)

Dean (References 1-4) and others present this mathematical solution technique entirely in terms of present-worth equivalence of discounted cash flows.[1]

[1] In this form, the desirability of an investment opportunity is indicated by the interest rate of return, which we may symbolize as ρ. If we denote the outlays incurred in the i-th time period as C_i, and the receipts as R_i, ρ may be determined by solving in

Although the mathematics for usual solutions are identical, the method of presentation of Hill (Reference 7) is followed here and in Reference 22. Equivalence of discounted cash flows is too far removed from the common-sense concepts of real project investment to allow an explanation, in its terms, of the problems discussed below. By analysis of the graph of cumulative cash position (the "CCP chart", Figure 1 following) association can be maintained with the realities of the status of investment and earning throughout the life of the project.

On Figure 1, time is the horizontal scale, and cash position is the vertical scale. Points in the "negative cash position" or "negative CCP" area, below the zero line, indicate investment by the company in the project. Points of "positive cash position" or "positive CCP", above the zero line, indicate profit and no residual investment by the company in the project. The terms

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the equation:

$$C_o + \frac{C_1}{1+\rho} + \frac{C_2}{(1+\rho)^2} + \cdots + \frac{C_t}{(1+\rho)^t} = R_o + \frac{R_1}{1+\rho} + \frac{R_2}{(1+\rho)^2} + \cdots + \frac{R_t}{(1+\rho)^t}$$

That is, the discounted value of the cost stream is set equal to the discounted value of the receipts stream. Here t is the final time-period in which the proposed investment has any effect on costs or receipts. The equation may be simplified by using the symbol $A_i = R_i - C_i$ to represent the net cash flow in the i-th period. Then ρ is found by solving:

$$A_o + \frac{A_1}{1+\rho} + \frac{A_2}{(1+\rho)^2} + \cdots + \frac{A_t}{(1+\rho)^t}$$

Note: Annual compounding used if period = 1 year.

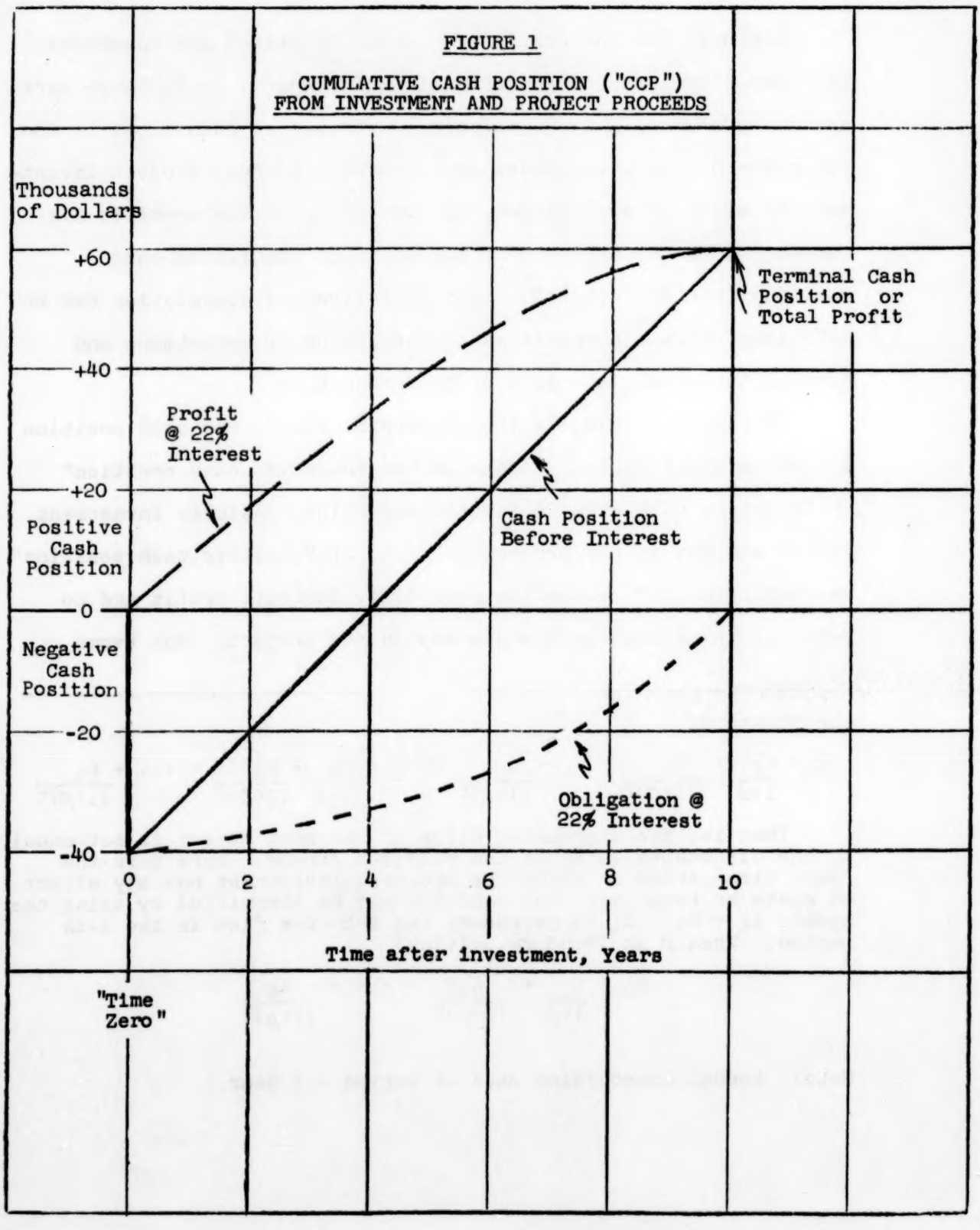


FIGURE 1
CUMULATIVE CASH POSITION ("CCP")
FROM INVESTMENT AND PROJECT PROCEEDS
Thousands of Dollars
+60
+40
+20
0
-20
-40
Positive Cash Position
Negative Cash Position
Profit @ 22% Interest
Cash Position Before Interest
Terminal Cash Position or Total Profit
Obligation @ 22% Interest
0
2
4
6
8
10
Time after investment, Years
"Time Zero"

"profit" and "residual investment" as used here will be clarified shortly.

Zero on the horizontal time scale is chosen as the start of production. The cumulative cash position of the company, and of the project, starts out as zero on the vertical scale; since the chart shows cumulative cash position, a single line indicates, from time zero on, the cumulative cash status of the project at each time indicated. Let us consider a sample case.

The solid line in Figure 1 represents the cumulative cash position of the company (or the project) before we consider interest. An instantaneous investment of $40,000 at time zero is shown, which yields $100,000 over the succeeding ten years.[1]

Income after time zero causes the solid line to turn upward. We may regard the income as being applied toward a reduction of remaining investment of the company in the project. Sometime before the end of the project, all investment is paid off, and the solid line crosses the zero cash position line. Since the chart is cumulative, the end of the solid line (10 years) shows a net profit of $60,000 ($100,000 - $40,000). All this is without consideration of interest.

The effect of interest on investment obligation is shown by the short-dash line; obligation is increased regularly by interest after time zero. (The term "obligation" throughout this paper

[1] For simplicity, complicating factors such as construction period, working capital, salvage value, etc., are omitted from this sample case. Reference 22 includes discussion of these items. All cases are calculated with continuous interest present-worth factors (Reference 22), although the mathematics of various footnotes is shown using annually compounded interest.

refers to the changing unpaid balance of principal plus interest less profits; cash income in each time period is used first to pay interest and any remainder is applied to retirement of principal.) The short-dash line indicates this obligation at an interest rate which will maintain some obligation throughout the project life, but reduce the obligation to zero just at the end of the estimated project life (in general, zero cash position after recovery of any working capital and land or salvage value). The compound interest rate which accomplishes this is referred to in this paper as the "interest rate of return" or the "solving interest rate". It has also been called the "true economic earning rate" (Reference 16) or the "internal rate of profit" (Reference 17). It is the interest rate paid by the project to the company for the use of company funds. All profits have been converted so as to be expressed as this interest rate, and the obligation of the project to the company, which has continued to be some real obligation throughout the project life, has been completely repaid. The method of solution, by trial and error, is described in the References cited above, and is indicated in the cash flows shown with Figure 1.[1]

[1] For the case of an initial cash outlay followed by a stream of receipts, it is simple to show that the rate λ which sets the final obligation equal to zero is the same as the rate ρ derived from setting the discounted cash flows equal to zero. In this case, ρ is determined by the equation:

$$C_o = \frac{R_1}{1+\rho} + \frac{R_2}{(1+\rho)^2} + \cdots + \frac{R_t}{(1+\rho)^t}$$

The obligation Θ_i in the time-period i may be defined sequentially

The profit (now expressed as interest rate) is the vertical difference between the solid and the short-dash line. The long-dash line shows this difference, the accumulation of interest payments to the company at the rate ρ throughout the project life. The identity of profit and interest payments at the end of the project life is indicated by the crossing of the two lines.

The Lender-Borrower Relationship

For analysis of profitability by interest rates, funds required by each project must be considered as loaned to it by the company. The project is considered a separate entity; it accepts funds which must later be repaid, with compound interest. The relationship under study is therefore close to that of the lender and the borrower in the case of a loan.

---

as follows:

$$\theta_0 = C_0$$

$$\theta_1 = C_0 (1+\lambda) - R_1$$

$$\theta_2 = C_0 (1+\lambda)^2 - R_1 (1+\lambda) - R_2$$

$$\vdots$$

$$\theta_t = C_0 (1+\lambda)^t - R_1 (1+\lambda)^{t-1} - R_2 (1+\lambda)^{t-2} - \cdots - R_{t-1} (1+\lambda) - R_t$$

The rate λ is that which sets θ_t equal to zero. Then, substituting for C_0:

$$\frac{R_1}{1+\rho} + \frac{R_2}{(1+\rho)^2} + \cdots + \frac{R_t}{(1+\rho)^t} = \frac{R_1(1+\lambda)^{t-1} + R_2(1+\lambda)^{t-2} + \cdots + R_{t-1}(1+\lambda) + R_t}{(1+\lambda)^t}$$

Therefore, $\rho = \lambda$.

A lender seeks investment opportunities which, for the use of his cash over a period of time, give him a profit over the life of the project. He wants a positive cumulative cash position at the end of a loan; the CCP chart for a loan repaid in equal installments (principal plus interest) would look like Figure 1.

The borrower of funds wants a source of funds in the near future, for which he is willing to pay a penalty in terms of total dollars, over the life of the loan. He seeks a positive cash position during the life of a loan, but is willing to end up in a negative cash position. A CCP chart could be drawn from his point of view, as shown in Figure 2, an inverted version of Figure 1.

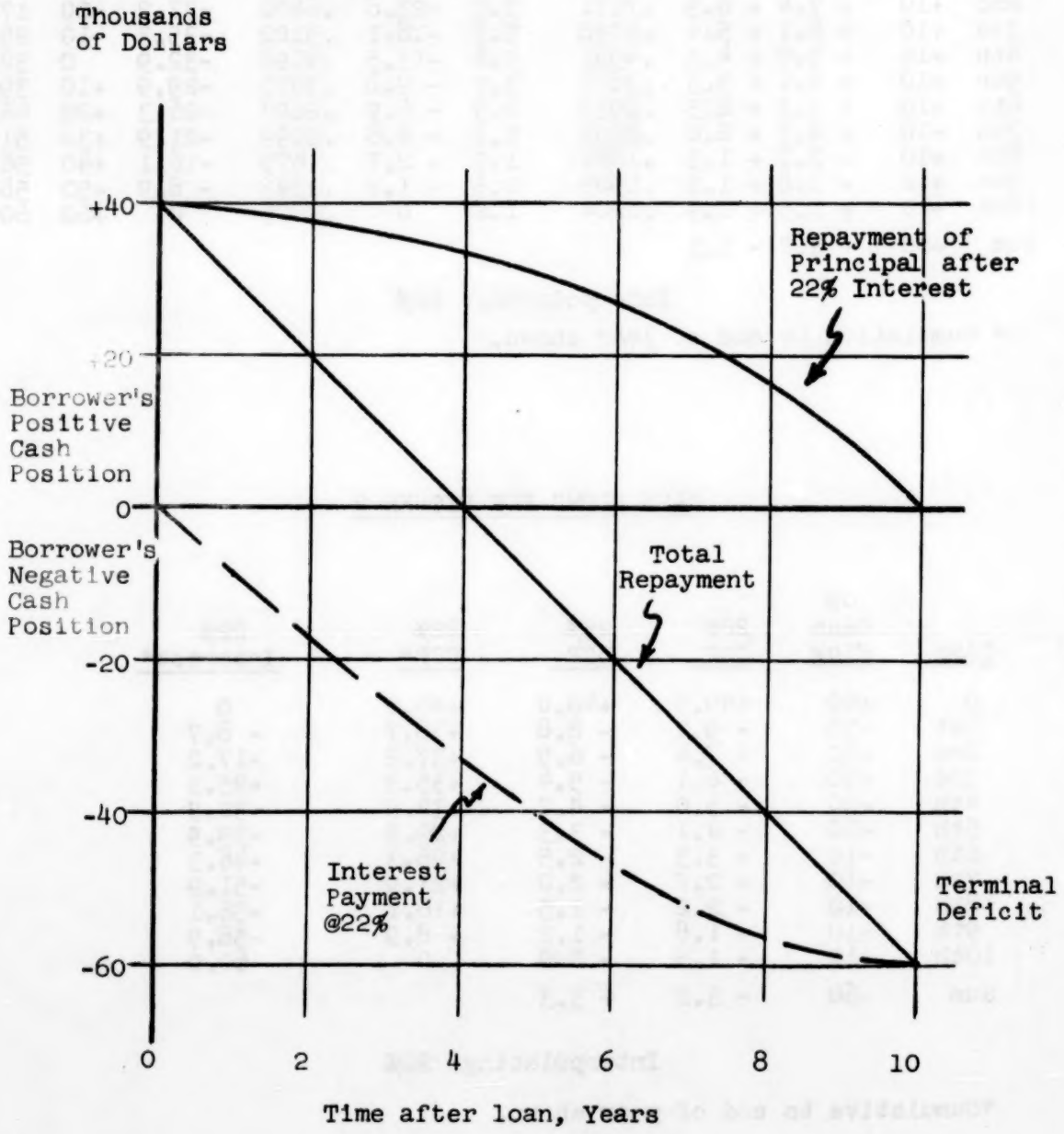
The solid line again shows the borrower's actual repayment pattern on the project, in equal installments. In order to see how much of each installment is interest, the payment can be compounded at various interest rates until the positive balance exists throughout the life of the project and is erased at the end of its life. The rest of the payments are then interest, as shown by the long-dash line. Note that solution is achieved by exactly the same technique as in Figure 1, and the same numerical interest rate is achieved as a solution. The mathematical technique used for solution by discounted cash flow for internal or interest rate of return does not distinguish between positive and negative cash positions, or between <u>earned</u> interest rates and <u>paid</u> interest rates.

The final cash position shown in Figure 2 is the position sought by a borrower; he expects to end up with an overall loss in

FIGURE 2

CUMULATIVE CASH POSITION FROM BORROWER'S VIEW

(Inverted CCP Chart)



CASH FLOWS FOR FIGURE 1

Time	0% Cash Flow	20% DCF	25% DCF	22.3% PW Factor	22.3% PW	22.3% CPW	22.3% PW Factor (Inst.)	22.3% CCP*	0% CCP*	22.3% Interest*
0	-40	-40.0	-40.0	1.0000	-40.0	-40.0	1.0000	-40.0	-40	0
1st	+10	+ 9.1	+ 8.8	.8963	9.0	-31.0	.8001	-38.7	-30	8.7
2nd	+10	+ 7.4	+ 6.9	.7171	7.2	-23.8	.6402	-37.2	-20	17.2
3rd	+10	+ 6.1	+ 5.4	.5738	5.7	-18.1	.5122	-35.3	-10	25.3
4th	+10	+ 5.0	+ 4.2	.4591	4.6	-13.5	.4098	-32.9	0	32.9
5th	+10	+ 4.1	+ 3.3	.3673	3.7	- 9.8	.3279	-29.9	+10	39.9
6th	+10	+ 3.3	+ 2.5	.2939	2.9	- 6.9	.2624	-26.3	+20	46.3
7th	+10	+ 2.7	+ 2.0	.2351	2.3	- 4.6	.2099	-21.9	+30	51.9
8th	+10	+ 2.2	+ 1.5	.1881	1.9	- 2.7	.1679	-16.1	+40	56.1
9th	+10	+ 1.8	+ 1.2	.1505	1.5	- 1.2	.1343	- 8.9	+50	58.9
10th	+10	+ 1.5	+ 0.9	.1204	1.2	0	.1075	0	+60	60.0
Sum	+60	+ 3.2	- 3.3							

Interpolating: 22%

* Cumulative to end of year shown.

CASH FLOWS FOR FIGURE 2

Time	0% Cash Flow	20% DCF	25% DCF	22% CCP*	22% Interest*
0	+40	+40.0	+40.0	+40.0	0
1st	-10	- 9.1	- 8.8	+38.7	- 8.7
2nd	-10	- 7.4	- 6.9	+37.2	-17.2
3rd	-10	- 6.1	- 5.4	+35.3	-25.3
4th	-10	- 5.0	- 4.2	+32.9	-32.9
5th	-10	- 4.1	- 3.3	+29.9	-39.9
6th	-10	- 3.3	- 2.5	+26.3	-46.3
7th	-10	- 2.7	- 2.0	+21.9	-51.9
8th	-10	- 2.2	- 1.5	+16.1	-56.1
9th	-10	- 1.8	- 1.2	+ 8.9	-58.9
10th	-10	- 1.5	- 0.9	0	-60.0
Sum	-60	- 3.2	+ 3.3		

Interpolating: 22%

*Cumulative to end of year shown.

return for his use of the funds over the period.

It is usual to consider the CCP chart only from the lender's view, by considering Figure 1 as presenting the borrower's position also, if read with signs reversed on the vertical cash-position axis. All further reference to the borrower's position will be by this means.

By definition, a lender would not put his money in a projected loan showing an estimated overall loss (before interest); this would result in the use of his funds without advantage to him. The lender seeks an initial negative balance and a terminal profit. Further, a borrower would not undertake a projected loan showing a net investment balance by him in the loan through most of the project life, since he would not be obtaining the funds which he as a borrower wished to have available for use.

Also, by definition, whenever the borrower holds no funds of the lender, he should pay no interest.

The Company-Project Relationship

The standard CCP vs. time chart (Figure 1) represents the cumulative cash position of the company as "lender" or allocator of funds to a project. The project's relationship must be differentiated from that of the borrower, however, since it is part of the company, and "interest" paid out by it (loss to a borrower) is truly profit both to the project and the company. Similarly, investments in the project (positive cash position to a borrower) are thought of as a negative cash position for the project. Thus, the cash position of both the company and the project are represented by the CCP chart with signs unchanged.

The company, like the lender, is looking for investment opportunities for its capital, and is interested in projects starting with investment (negative cash position) and ending up (zero interest) with a positive cash position, or overall profit. The interest rate of return is a method of comparing this profit to the investment (including interest obligation) by determining average interest rate paid on total funds invested, the "obligation" as previously defined.

Return on Investment

Profitability can best be expressed as a return on investment, since profits are then related to the funds tied up in creating and maintaining the source of the profit. Any relation expressing return on investment therefore assumes that there are funds tied up for all of a given period. When there are no longer company funds invested, profits can no longer be expressed as return on investment.

The usual interest rate of return calculation meets this requirement on a usual investment like Figure 1, since at the solving interest rate, found by trial and error, the short-dash line in Figure 1, representing the obligation or investment including interest, remains below the zero line for the entire life of the project, rising just to zero at project termination. References 7 and 22 show that the interest rate of return is equal to the average profit, over the entire project life including construction, divided by the average value of the obligation including interest. Profits are removed from the project to the

company, and no assumption need be made regarding their reinvestment.[1]

Only revenue from the project in question is considered in determining interest rate of return. Although funds made available as profit or depreciation early in the life of a project are available to the company for reinvestment, such reinvestment would require other investment opportunities which must be justified separately on their own merits. Therefore, no compounding of funds made available during a project (positive CCP) is felt to be a valid part of the justification for the project.

[1] This conflicts with statements by Happel (Reference 6) and Winn (Reference 22). However, continuing the nomenclature of the previous footnote, the initial obligation equals the initial investment.

$$\theta_o = C_o$$

Instantaneously, at the end of the first year, the obligation is increased by interest and reduced by the receipts.

$$\theta_1 = C_o + \lambda C_o - R_1$$

No compounding of receipts is involved up to this point. At the end of the second year, the obligation from the end of the first year is again increased by interest and reduced by the receipts.

$$\theta_2 = \theta_1 + \lambda \theta_1 - R_2$$

Again, no compounding of receipts is involved. Eliminating θ_1, of course, results in an expression which contains the product of receipts R, times interest, $(1 + \lambda)$.

$$\theta_2 = C_o (1+\lambda)^2 - R_1 (1+\lambda) - R_2$$

However, this term is negative and indicates only that the first-year obligation θ_1 does not include first-year receipts R_1. This is inherent in the mathematical demonstration but in no way means that receipts R_1 are compounded.

Compounding is applied only to a project obligation to the company, to determine what interest rate will express the relationship of profit and investment for all cash flows in question.

FALSE OR MULTIPLE SOLUTIONS

Periods of Loss

Some worthwhile projects will show a period of loss during their life. These usually occur at the start of the project, before markets for the product permit operation above the break-even point. Initial periods of loss will not affect the propriety of the standard discounted cash flow method of solution; as shown in Figure 3, the CCP chart after interest (short-dash line) still maintains an obligation (negative CCP) throughout the project life.

However, periods of loss may occur later in a project's forecast life. A lease may be required, part of which must be paid off after the completion of income from this project. In an oil well, the later years may show a reduced output of oil, still sufficient to pay direct costs and make some contribution toward covering fixed expenses, but not truly profitable. After such a low-yield period, a secondary recovery project could be included in the forecast, so the loss period could be part way through the project life.

When such losses occur later in a project life, there is a real danger of false or multiple solutions for interest rate of return by the standard techniques. For example, Figure 4 presents an example such as might be encountered if a lease payment were

FIGURE 3

CUMULATIVE CASH POSITION ANALYSIS OF INVESTMENT WITH INITIAL LOSS PERIOD

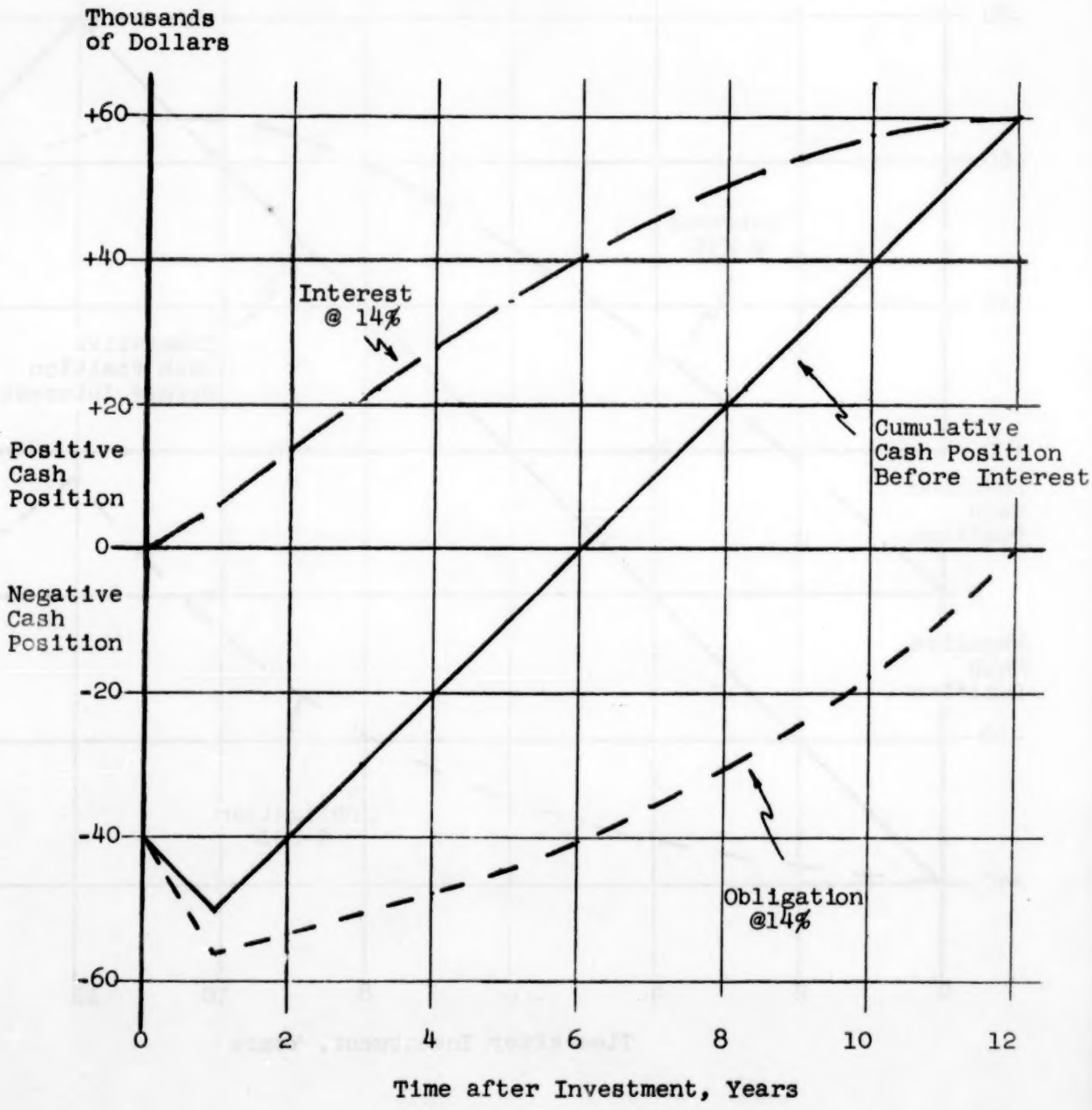
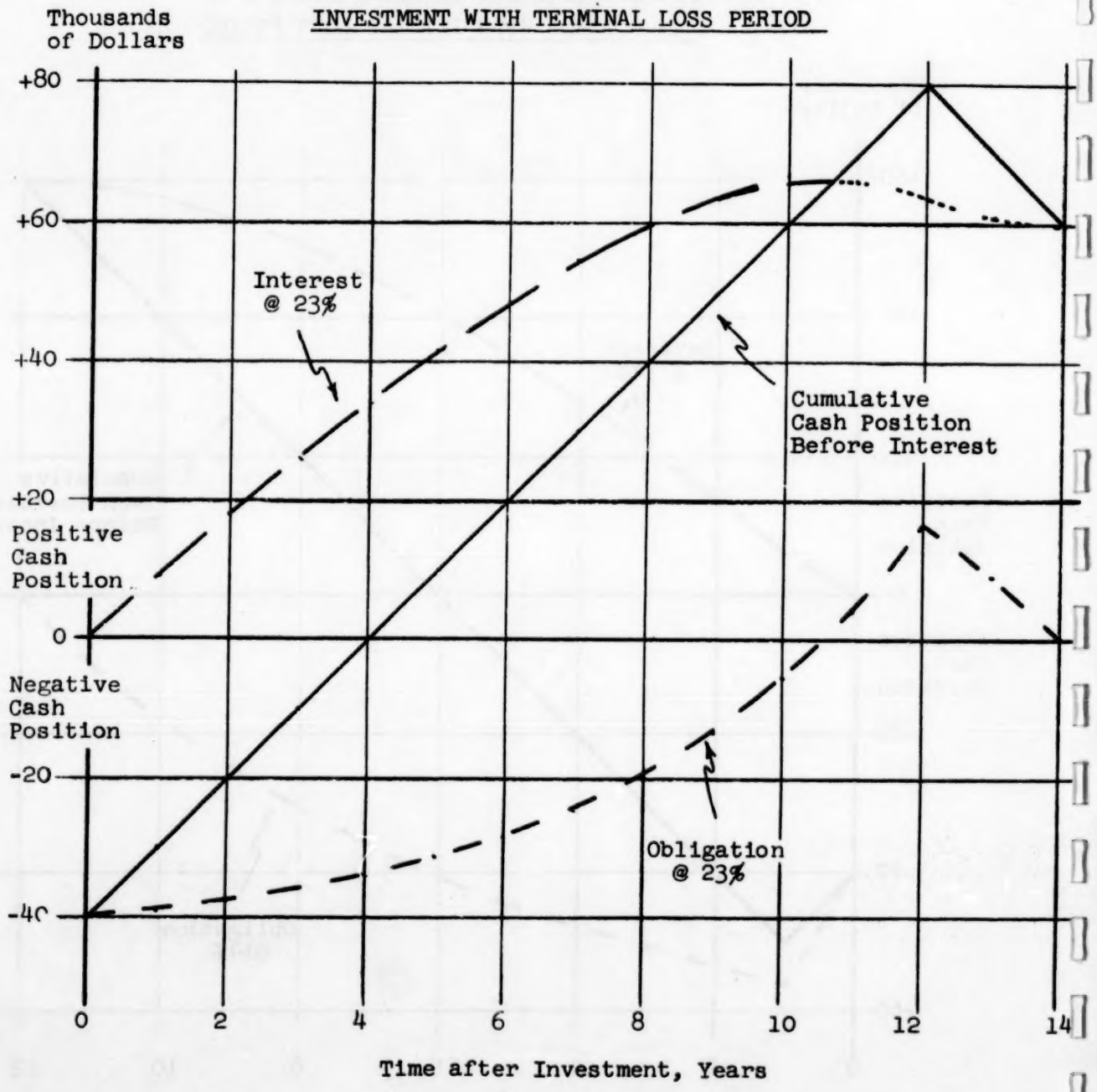


FIGURE 4

CUMULATIVE CASH POSITION ANALYSIS FOR INVESTMENT WITH TERMINAL LOSS PERIOD



CASH FLOWS FOR FIGURE 3

Time	0% Cash Flow	10% DCF	15% DCF	14% CCP*	14% Interest*
0	-40	-40.0	-40.0	-40.0	0
1st	-10	- 9.5	- 9.3	-56.6	6.6
2nd	+10	+ 8.6	+ 8.0	-54.4	14.4
3rd	+10	+ 7.8	+ 6.9	-51.7	21.7
4th	+10	+ 7.0	+ 5.9	-48.6	28.6
5th	+10	+ 6.4	+ 5.1	-45.1	35.1
6th	+10	+ 5.8	+ 4.4	-41.0	41.0
7th	+10	+ 5.2	+ 3.8	-36.2	46.2
8th	+10	+ 4.7	+ 3.2	-30.7	50.7
9th	+10	+ 4.3	+ 2.8	-24.4	54.4
10th	+10	+ 3.9	+ 2.4	-17.2	57.2
11th	+10	+ 3.5	+ 2.1	- 9.2	59.2
12th	+10	+ 3.2	+ 1.8	0	60.0
Sum	+60	+10.9	- 2.9		

Interpolating: 14%

*Cumulative to end of year shown.

CASH FLOWS FOR FIGURE 4

Time	0% Cash Flow	20% DCF	25% DCF	23% CCP*	23% Interest*
0	-40	-40.0	-40.0	-40.0	0
1st	+10	+ 9.1	+ 8.8	-39.0	9.0
2nd	+10	+ 7.4	+ 6.9	-37.8	17.8
3rd	+10	+ 6.1	+ 5.4	-36.1	26.1
4th	+10	+ 5.0	+ 4.2	-34.2	34.2
5th	+10	+ 4.1	+ 3.3	-31.7	41.7
6th	+10	+ 3.3	+ 2.5	-28.5	48.5
7th	+10	+ 2.7	+ 2.0	-24.5	54.5
8th	+10	+ 2.2	+ 1.5	-19.7	59.7
9th	+10	+ 1.8	+ 1.2	-13.1	63.1
10th	+10	+ 1.5	+ 0.9	- 4.8	64.8
11th	+10	+ 1.2	+ 0.7	+ 4.9	65.1
12th	+10	+ 1.0	+ 0.6	+16.8	63.2
13th	-10	- 0.8	- 0.4	+ 9.6	60.4
14th	-10	- 0.7	- 0.3	0	60.0
Sum	+60	+ 3.9	- 2.7		

Interpolating: 23%

*Cumulative to end of year shown.

required beyond the life of the expected income. The result would be a loss for the last years of the forecast. By the standard discounted cash flow solution technique, the cumulative cash position after interest (short-dash line) shows a period where there is no investment of funds by the company in the project (positive CCP, short-dash line above zero line). The company must hold net funds in reserve in order to pay off the lease when it comes due. The obligation (negative CCP) is reduced to zero before the end of the project life, the CCP rises to a positive amount, and returns to zero again at the end of the project life.

During such a positive CCP period, the discounted cash flow technique applies the same interest factor as to the rest of the life, but it is not applied to an investment which is to be discounted back to zero time. Instead, it is a positive balance in the hands of the company, which the technique compounds at the solving interest rate -- although no "investment" remains and although funds received during this period must be held liquid by the company to pay off the lease.

This compounding does not seem warranted. The project is earning no interest on these funds, and the company cannot reinvest them even at their average earning rate, since they must be held liquid. It seems more realistic to set up a mathematical technique so that such "positive CCP balances" (occurring after a solving interest rate is applied to a cash flow) may be compounded at no more than a short-term interest rate.

Delayed Investments

An investment which is forecast to occur after the start of income from a project causes a decrease in the cumulative cash position of the project as of that time, which may result in a similar unrealistic compounding of positive balances. It is clear that any terminal investment (disposal of dangerous properties, etc.), or any investment after the original investment has been repaid from project income, will show a positive cumulative cash balance before interest (solid line) between two negative areas of obligation. These positive areas may remain after interest is applied, and will be compounded at the solving interest rate. They will therefore also give false solutions by the usual discounted cash flow technique.

Determining Occurrence of False Solutions

A sure method for determining such compounding of positive balances in a discounted cash flow solution, whether due to a period of loss or delayed investment, is to look for a positive present worth at any time during the project life. (See years 11-13, CCP, Cash Flows for Figure 4.) The following procedure is recommended:

> Whenever a loss occurs in an investment project (other than losses immediately after the start of income) or an investment occurs after revenue has begun to accrue, cumulate the present-worth column at both trial interest rates which bracket zero present worth (CPW column in Figure 1). If either or both show a positive present worth before the project termination, the indicated solution may not be a correct one. The period of positive balance should be determined by plotting the cumulative cash position as in Figure 4, at various interest rates (see the Appendix for this procedure). By trial and

error, an interest rate can then be found which will give a zero present worth without compounding the positive balance, or by compounding it at the short-term interest rate. Either is considered a more correct solution.

A special case of this rule is a final negative cash transaction on a project, or a series thereof, such as Figure 4, which will always give a false interest rate of return solution. However, the correction is fairly easy to make in this case, since the end of the period of positive balance is known to be the end of the project. It is then easy to determine that time during the project, after which all earnings must be set aside either inactive or in liquid holdings, to pay for the subsequent loss. The interest rate of return is then found by normal means, with this earlier time as a pseudo-terminal. Note that this rate of return is earned only to the pseudo-terminal, not for the full life of the project.

Another special case is a project with an initial positive balance, as described below under "Multiple Solutions". These also always give incorrect answers by usual methods, due to compounding of the initial positive balance. Such projects can be treated in a parallel manner, to find a pseudo-start when net investment begins.

Trial and error is the usual discounted cash flow method of solution; two interest rates are found which bracket the zero present worth point, and interpolation is used to find the interest rate for zero present worth. Therefore, even in solution of normal problems, the lower interest rate solution gives some

positive terminal balance, which is a compounded balance. However, if interpolation is over a reasonably small percentage range, this error is not felt to be significant.

Acceleration Problems

Interest rate of return has been used to evaluate the incremental investment needed to accelerate a fixed income. The total "income" from such an incremental project occurs over the acceleration period only and a "loss" therefore occurs before the end of the equipment life, at the time when the same income would have been obtained under the other alternative.

Acceleration problems are a special case, since only incremental investments are considered, and the only "incomes" are earlier receipt of funds. As discussed by Solomon (Reference 17), these can only be justified if the earlier receipt of funds makes funds available which can be utilized in other projects and compounded sufficiently to make up for the overall loss which exists in every acceleration project. Because of the incremental nature of these projects, one statement (in "Periods of Loss", above) does not apply. It was said there that positive balances in the hands of the company must be held idle or liquid for use later in the project. In an acceleration project, such funds would be available for investment by the company elsewhere, and available sooner, which is their value to the company. Note that they are always loss projects before interest, and therefore cannot be acceptable without this compounding. However, any investment of these funds is in another project and it seems unfair to credit

those earnings to this project.

No solution to the problem of acceleration projects is proposed here. As concluded by Solomon, interest rate of return does not seem capable of evaluating such problems.

Multiple Solutions

Multiple answers have already been reported for return on investment problems solved by the discounted cash flow method (References 11, 17, and 20). No explanation is given of the meaning of the two rates, and the recommendation presented there is to use another method for solution of such problems.

But the multiple solutions they find are believed to be merely special cases of the false solutions just discussed. Such problems are unusual, but may occur. A project which enables the borrowing of specific funds before the start of construction could be considered to start with a positive balance. A purchase of a going company where all payment was delayed, or tax benefits therefrom, might also cause a project to provide funds before the start of investment. Lease arrangements permitting subsequent purchase will also show, in some cases, considerable income before the major investment. Such a project would have a CCP chart with the characteristics of Figures 5A and 5B.

If usual discounted cash flow methods are used to evaluate such a project, the initial period of positive balance is compounded at any interest rate applied, as shown in the cash flows beside Figure 5. Because of the interrelationships among interest factors, the worth of this type of project at the end of the

project life, when subjected to increasing interest rates, usually looks like Figure 6, which represents the same case presented in Figure 5.

Note that the two solving interest rates of Figure 5 (31% and 71%) are crossings of the zero-terminal-worth line in Figure 6. Because of compounding of positive balances, neither crossing is felt to be the correct answer.

For two crossings of the zero CCP line, there may be two points of zero terminal worth, as shown; only one, if the curve is tangent to zero; or none, if the compounding of the positive balance never lets the terminal worth fall to zero. The mathematics of multiple solutions are complicated and intriguing, but they are only a part of the broader and more important problem of false solutions.

Since all of the multiple solutions are false and since false solutions may occur whether the number of solutions is single or multiple, this is not considered a fruitful area for further study.

If positive balances are not compounded, multiple rates of return are not encountered. If no positive CCP balances are compounded, there is never a point of inflection (change of direction) in the plot of terminal worth of obligation vs. interest rate (corresponding to Figure 6).

CONCLUSION

Periods of loss after the start of income from a project, or investments timed after the start of income, are shown to produce

FIGURE 5A

CUMULATIVE CASH POSITION ANALYSIS OF MULTIPLE SOLUTIONS IF INCOME PRECEDES INVESTMENT

A. Solution at 31% Interest

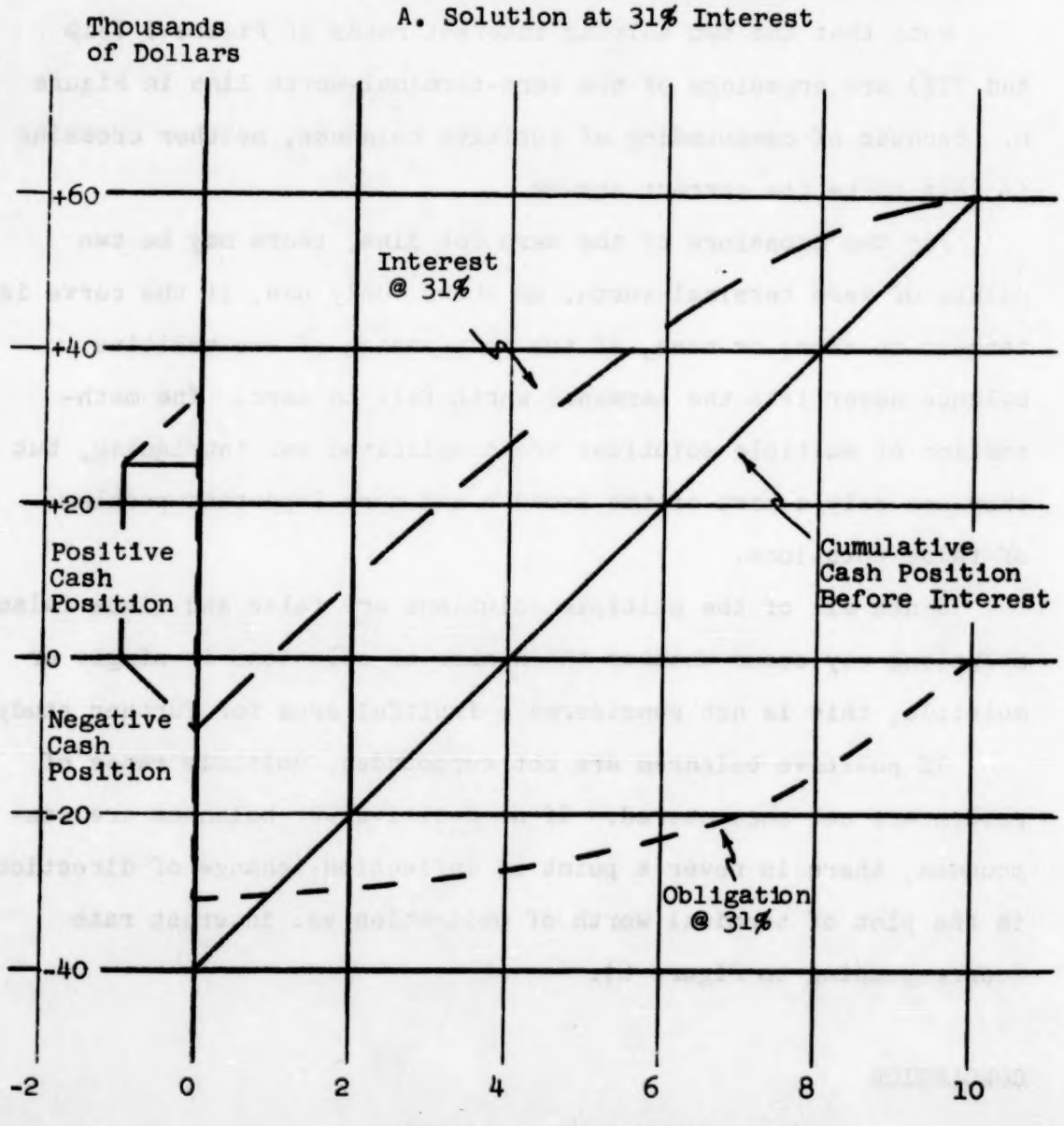
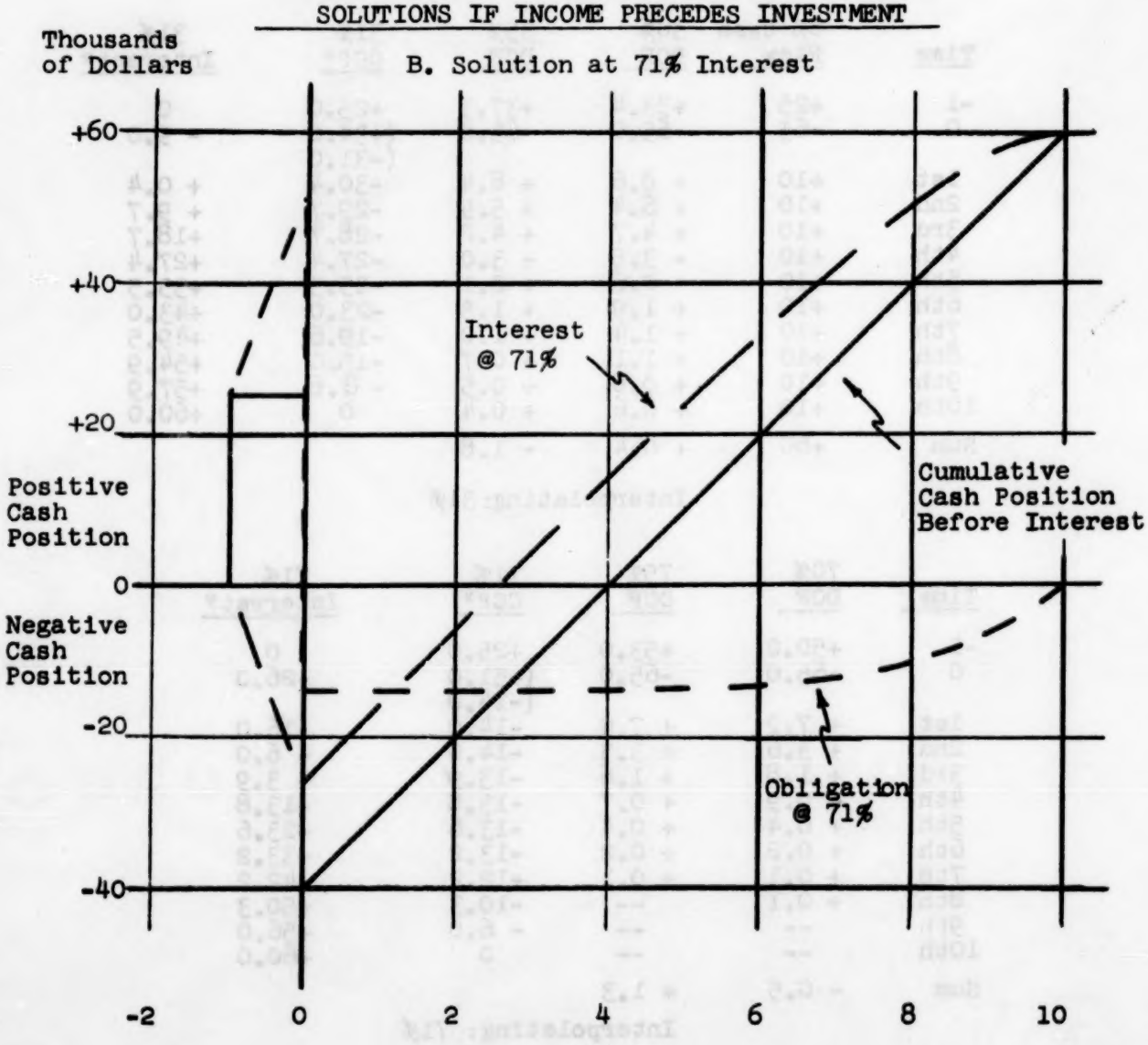


FIGURE 5B

CUMULATIVE CASH POSITION ANALYSIS OF MULTIPLE SOLUTIONS IF INCOME PRECEDES INVESTMENT



CASH FLOWS FOR FIGURE 5

Time	0% Cash Flow	30% DCF	35% DCF	31% CCP*	31% Interest*
-1	+25	+33.4	+37.3	+25.0	0
0	-65	-65.0	-65.0	(+34.0	- 9.0
				(-31.0	
1st	+10	+ 8.6	+ 8.4	-30.4	+ 0.4
2nd	+10	+ 6.4	+ 5.9	-29.7	+ 9.7
3rd	+10	+ 4.7	+ 4.2	-28.7	+18.7
4th	+10	+ 3.5	+ 3.0	-27.4	+27.4
5th	+10	+ 2.6	+ 2.1	-25.5	+35.5
6th	+10	+ 1.9	+ 1.5	-23.0	+43.0
7th	+10	+ 1.4	+ 1.0	-19.6	+49.5
8th	+10	+ 1.1	+ 0.7	-15.0	+54.9
9th	+10	+ 0.8	+ 0.5	- 8.6	+57.9
10th	+10	+ 0.6	+ 0.4	0	+60.0
Sum	+60	+ 0.4	- 1.8		

Interpolating: 31%

Time	70% DCF	75% DCF	71% CCP*	71% Interest*
-1	+50.2	+53.0	+25.0	0
0	-65.0	-65.0	(+51.0	-26.0
			(-14.0	
1st	+ 7.2	+ 7.0	-14.0	-16.0
2nd	+ 3.6	+ 3.3	-14.0	- 6.0
3rd	+ 1.8	+ 1.6	-13.9	+ 3.9
4th	+ 0.9	+ 0.7	-13.8	+13.8
5th	+ 0.4	+ 0.4	-13.6	+23.6
6th	+ 0.2	+ 0.2	-13.2	+33.2
7th	+ 0.1	+ 0.1	-12.2	+42.2
8th	+ 0.1	--	-10.3	+50.3
9th	--	--	- 6.0	+56.0
10th	--	--	0	+60.0
Sum	- 0.5	+ 1.3		

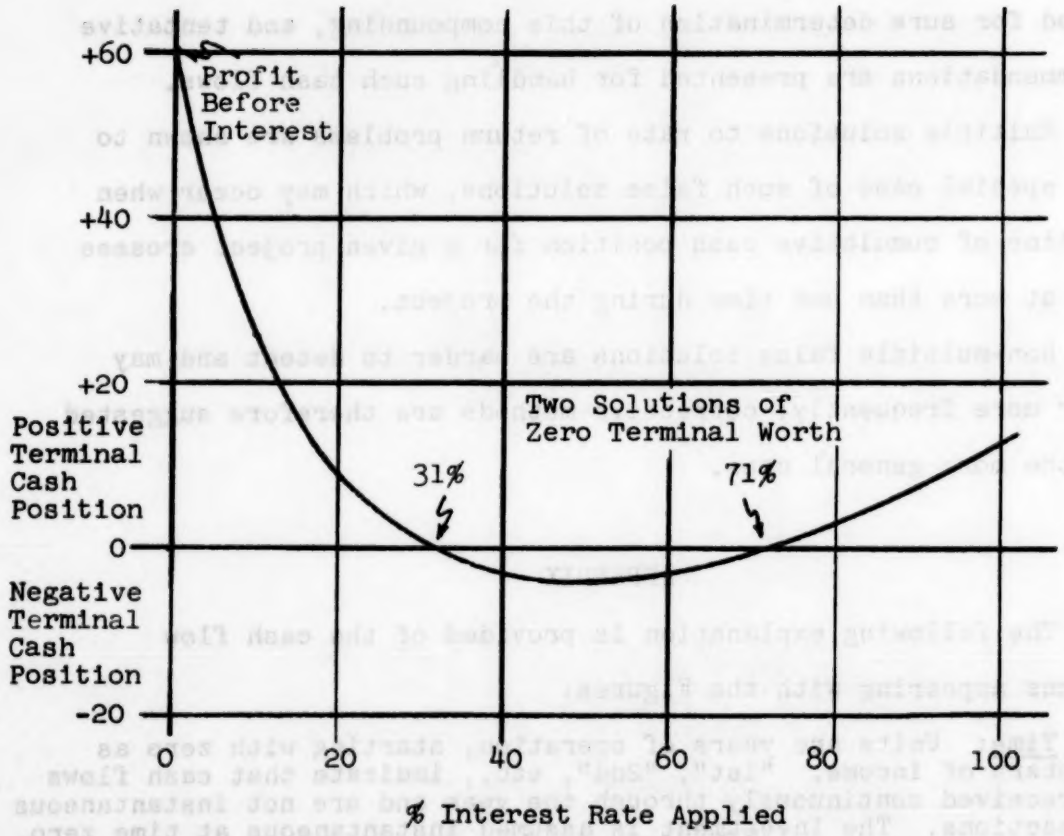
Interpolating: 71%

*Cumulative to end of year shown.

FIGURE 6

EFFECT OF INCREASING INTEREST RATE ON
TERMINAL WORTH IF INCOME PRECEDES INVESTMENT

(Case from Figure 5)



cash flow patterns sometimes resulting in compounding instead of discounting when "discounted cash flow" techniques are applied. Terminal loss or terminal investment transactions always result in such compounding. Acceleration projects, being overall loss projects, always result in such compounding. A method is described for sure determination of this compounding, and tentative recommendations are presented for handling such cash flows.

Multiple solutions to rate of return problems are shown to be a special case of such false solutions, which may occur when the line of cumulative cash position for a given project crosses zero at more than one time during the project.

Non-multiple false solutions are harder to detect and may occur more frequently; corrective methods are therefore suggested for the more general case.

APPENDIX

The following explanation is provided of the cash flow columns appearing with the Figures:

Time: Units are years of operation, starting with zero as the start of income. "1st", "2nd", etc., indicate that cash flows are received continuously through the year and are not instantaneous transactions. The investment is assumed instantaneous at time zero.

0% Cash Flow: These are the actual cash transactions at the time or during the year shown. A cumulation of these gives the coordinates of the solid line "CCP before interest". The final sum is the net profit from the venture over its life.

"20% DCF", "25% DCF", etc.: Discounted cash flow (DCF) columns are the product of the "0% Cash Flow" and the applicable interest factor for cash flows received in the manner shown, at the time shown, and at the rate shown. Continuous interest tables were employed (see Reference 22). A method for choosing a starting interest rate for this trial and error solution is described

in Reference 22. The final sum shows the present worth of the terminal value of the project after payment of the indicated interest rate.

Interpolating: Solution by this usual trial and error technique determines the interest rate which would discount the cash flows so as to cumulate to a zero CCP. This is the "interest rate of return" or "solving interest rate". It is obtained by interpolating between the cumulated balances bracketing zero terminal worth.

A column can then be developed to show the discounted cash flow (DCF) at the solving interest rate. This column is omitted for brevity, except in Figure 1, as is the "cumulative present worth" column, which cumulates to zero.

"Solving % CCP" (Cumulated to End of Year Shown): This column presents the cumulative value of the project as of the time shown, as differentiated from the "cumulative present worth" which is calculated as of time zero. One method for calculation is shown in Reference 22. It determines the coordinates of the short-dash line, or "obligation", as defined on page 5-6. It can also be calculated by dividing the cumulative present worth by the proper interest factor, at the solving rate, for the year indicated.

"Solving % Interest" (Cumulated to End of Year Shown): This represents the interest accrual on the unpaid obligation, the long-dash line in the figures, and is the vertical difference between the solid and short-dash lines. It is the portion of the obligation accrued due to the interest rate applied. It is the difference between the "0% Cash Flow" column and the "Solving % CCP" column.

ACKNOWLEDGEMENTS

The creative participation of A. G. Bates, J. Hirschleifer, and R. J. Reilly, in the preparation of this paper, is hereby acknowledged.

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Review Article

MODERNIZING THE BREAK-EVEN ANALYSIS *

By

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I.

Classification of business costs in relation to an activity function is a fundamental feature of the techniques of modern profit planning and control. One of its best known applications is the break-even chart. Raymond Villers calls it a device for over-all profit optimization based on a mathematical study of the cost to sales relationship, the first presentation of what we would today call a mathematical model (1).

The popularity of the break-even chart among business men and analysts of business data is mainly the result of its apparent simplicity. The average businessman, who is generally unwilling to think and reason in terms of abstract functional relationships, can be made to understand the simple graph representing the break-even concept and to work with it in his everyday decision making.

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However, this apparent simplicity has to be bought at a substantial sacrifice. Numerous important factors affecting the validity of the data which are used in drawing up the break-even chart either are disregarded altogether or are made subject to the assumption of "other things remaining unchanged" (2).

One of the major shortcomings of the break-even chart is its static nature. It reflects conditions at a particular moment on the basis of those conditions existing on that date, like a snapshot. This limits its usefulness for evaluating different alternatives and conditions. It is a substantial handicap in an economy like the present one, in which changes are frequent. In order to be relevant, the data underlying the break-even chart must be brought up-to-date at frequent intervals. The chart then has to be redrawn each time to provide meaningful correlations for management planning. This is a time consuming and expensive chore which tends to offset the gains resulting from the chart's original simplicity.

Raymond Villers, Rautenstrauch's close associate and a thorough student of the break-even chart and its practical applications, has been aware of this problem. He knows that " to compute the base data in a careful manner takes time and effort. It is worthwhile doing this from time to time but it cannot be repeated too often." Therefore, he is now suggesting a new method whose essential purpose is to avoid the repetition of these computations at too frequent intervals (3). It should minimize the disadvantages inherent in the static break-even chart.

It is the purpose of this review to analyze this new method

and to determine its value in improving the applicability of the break-even chart concept.

II.

The author calls his method of differential profit control an attempt to provide the necessary tool to analyze a whole situation through the use of a mathematical model which truly reflects the complexity of the problems of profit control. This model is a further development of the break-even chart which Villers already calls a mathematical model, as indicated above.

The break-even chart provides, first, a linear equation which defines the cost to sales relationship, within a certain range of variation of the volume of production which remains valid as long as the conditions of operations remain unchanged. Then, when this range of variation of the volume of production is exceeded and/or when the conditions of operations change, the chart provides the basis from which the impact of a change can be measured.

The new method combines in a single equation (mathematical model) both the linear equation and the structure within which computations can be made in a comparatively simple manner to measure the impact of a change. This is accomplished by incorporating into the original break-even equation the functions which represent the changes in constant costs (Δa) and in the ratio of variable costs to sales (ΔC), since the time when the chart was first prepared.

Thus the original equation

$$y = a + Cx$$

is changed to

$$y = (a + \Delta a) + (C + \Delta C)x$$

The changes that may require an adjustment of the break-even chart are classified and analyzed as follows:

The constant expenses cannot change without a managerial decision authorizing a change. The variable expenses do change, more or less automatically, as sales vary, but the cost to sales ratio cannot change substantially unless conditions of operations are altered by a managerial decision or there is a change in the sales mixture.

All these changes can be automatically detected and subsequently evaluated by merely establishing a routine procedure providing for their recording and the computation of their impact. They can be classified as follows:

1.1 Differences due to a change in the value of the original elements of constant costs.
1.2 New elements of constant costs that have appeared after the time of the original recording.

2.1 Differences due to a change in variable costs which are related to a change in the structure of the original elements of variable costs.
2.2 New elements of variable costs that have been introduced.

3. Changes in sales volume if the variable costs are not all related to the same sales volume.

4. Differences due to a change in the sales mixture.

The method of differential profit control can essentially

serve the following purposes:

1. The determination of a reliable standard to which the actual performance can be compared.

2. The budgeting of future expenses in relation to anticipated sales.

3. The evaluation in quantitative terms of the relative worth of alternatives under consideration.

4. The preparation of a differential break-even chart which indicates trends at different dates.

The fundamental characteristic of the method of differential profit control, according to the author, is essentially intended to reduce the work involved in break-even analysis, by limiting it, whenever possible, to a computation of differences. The differential break-even chart can be used to show the impact of any change, whether an actual one or one that is under consideration, by merely plotting the trend corresponding to the situation that has been or would be created by the change. The new trend is determined by using either the previous or the present trend as the basis from which to start and evaluate any expected difference.

III.

Since the author emphasizes the purpose of his suggested method as a means to reduce computational work, the following discussion will concentrate on the question of whether or not he has accomplished this aim. Although a major portion of his paper is devoted to the benefits that can be derived from the break-even chart itself, this matter will not be taken up in the present context.

The analysis of the components of the differential between two time periods is certainly valuable; it points out the three basic factors of cost, sales or production volume, and sales or production mix. (The last item, particularly, is often overlooked.) It is also useful to distinguish between changes in existing costs and the addition of new ones, in determining cost differentials between time periods.

Focusing attention on differentials, rather than comparison of totals, is always advisable as a means of applying the principle of exception as a tool of managerial control. But the value of the differential as a clerical cost saving is more doubtful.

It is surprising that an expert with such a wide experience fails to give any attention to the technical problem of accumulating the data which are needed for determining the differential, for this problem seems to be substantial if not forbidding. The number of changes affecting the data to be incorporated into the break-even chart could be very large even for a medium-sized company. If the company has a system of machine accounting, the relevant data could be accumulated periodically, although the machine time and expense involved would be far from negligible. Without such a system, the difficulties would be very great.

Probably it would be simpler and cheaper to get the "updated" information from a current trial balance, and then either prepare a new break-even chart or draw new trend-lines into the existing chart. But the latter procedure could not be repeated too often without making it difficult to interpret the chart.

Perhaps the author has in mind that only the larger changes

should be picked out and summarized in the differential, and that a complete total of the changes is not obtained. This leads to the question of what margin of error should be accepted in a break-even chart.

The simplifying assumptions used in computing break-even charts must affect the accuracy of the data and the significance of the conclusions drawn therefrom. For instance, most cost classifications are not truly variable or fixed (constant), but are semi-variable; they fluctuate with the activity factor but not at a steady rate (which is, of course, zero for the constant costs). The break-even chart disregards that undeniable fact. This, in turn, will have its effect on the interpretation of the differential between two periods. In part, it may merely reflect the effects of the simplifying assumptions.

It is doubtful whether the new method suggested by Dr. Villers makes any significant contribution either toward an expansion of the scope of the break-even chart or as a means to eliminate the effects of its static character by simplifying the computation of the necessary up-dating figures. It does not remove any of the existing shortcomings of the chart.

The situation is similar to the distinction between a fixed and a flexible budget. The latter eliminates the disadvantages of fixed data in the light of changing conditions, by computing a series of data within an expected range of fluctuations. Such a technique was recommended for the break-even chart by Joel Dean some years ago (2). Naturally, it is more complex and time consuming than the original simple break-even chart, just as the

flexible budget is more difficult to prepare than the fixed one. If usefulness to management is taken as a yardstick for making a choice (as it certainly should be), rather than mere expense consideration, there can be no doubt about the proper selection.

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Review Article

PROFITABILITY INDEX FOR INVESTMENTS *

By
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In his enthusiasm for the "PI" approach to capital expenditure problems, Mr. Reul seems to have overstated the case for this particular method and understated the case for alternative methods.

In the subheading, under the title of this article, are listed four advantages claimed for the "PI" approach, namely: that it "applies to any capital expenditure"; that it "involves simple calculations"; that it "offers a uniform base for comparisons"; and that it "requires no arbitrary assumptions". I submit that at least two, and possibly three of these statements are false, and that the fourth "advantage" is shared by some of the systems criticized.

If falseness is a matter of degree, probably the most false of the claims would be that "no arbitrary assumptions are required". Now I am not sure just what makes an assumption arbitrary, but I am sure that any mathematical model used to describe a real-life situation reqûires a number of assumptions. Reul himself, in his article, states several which are required for the "PI" method. (Incidentally, why not give it the name given by its

* RAY I. REUL: Harvard Business Review, Volume 35, Number 4, pp. 116-132, July-August 1957.

first popularizer, Joel Dean, and call it the "Discounted Cash Flow" method?) Others not mentioned include a constant price level, an unchanging tax structure, and what might be called an unchanging level of technology -- that is, that <u>better</u> ways of doing the same job will not become available in the future. I will comment more on this last matter later, in examining Mr. Reul's criticism of other methods.

The second subheading I would question is that this method "applies to any capital expenditure". The author has failed to show how it can be applied to an equipment replacement decision, although such a demonstration would seem to be required.

The "PI" method, or "Discounted Cash Flow" approach, is, quite simply, a present worth calculation in which the interest rate is the unknown. As such, it is quite logical to use in examining the rate of return earned on a particular investment, independent of any other choice that might be made.

But in a replacement decision this is not the case. In a replacement decision, the relative economy of keeping an existing asset must be compared to that of acquiring a new piece of equipment. In making this comparison, the life of each asset should, in general, be considered as that which will minimize its average annual cost. Now this will almost always result in differing lives for the existing and new equipment. How, then, can a present worth calculation be used? Surely the present worths of the expenditures incurred in two differing periods of time are not comparable. Nor can this difficulty be overcome by assuming that a new piece of equipment, just like that presently proposed and

at the same cost (more arbitrary assumptions), will be purchased at the time of the eventual retirement of the existing asset. For then, in order that both alternatives may have common time periods (as required by the present worth approach) one or the other would have to be kept for a period other than its most economic life.

If, then ,there is no simple way in which a present worth approach can be used to compare the desirability of alternatives having differing lives, it would seem incumbent upon the proponent of such a method to demonstrate that this is not an insurmountable obstacle, before claiming that such a method "applies to any capital expenditure".

Furthermore, the present worth approach seems ill-adapted to utilization of the principle that each additional increment of investment should pay its own way. In other words, if, by a small expenditure you could get <u>most</u> of the advantage of a much larger expenditure, the larger expenditure might not be wise even though the over-all rate of return it offers is quite satisfactory. Why? Because it might be possible to invest the additional capital required by the larger investment in a way that offered a higher return than the difference between that of the larger and smaller investments.

My quarrel with the claim that "PI" offers a "uniform base for comparison" may be a semantic one. But does a "PI" of, say 25%, mean the same thing in all cases? Would it indicate the same course of action if the project under consideration were a very risky one, as it would in a low-risk situation? Would it mean the

same thing as a measure of the profitability of investing in a general purpose tool, such as a punch press, on which we have a long history to base future assumptions, as it would as a measure of the return on, say, the establishment of a transistor plant?

This leads naturally to a comment on the objections raised by Mr. Reul to the average annual cost method. But first, we should perhaps note that the fourth advantage claimed for "PI" -- that of simple calculations -- is at least equally true of the annual cost and MAPI methods.

In commenting on the annual cost method, it is stated that "This method . . . requires the assumption of one minimum acceptable interest rate. Have you ever tried to get executives in your company to agree on such a rate? How can they, when the acceptable rate will vary with the risk involved?"

There is absolutely nothing in the annual cost approach that could be construed as requiring the assumption of a single, minimum attractive rate. Some of the users of this method have imposed this restriction upon themselves; but they might have done the same had they used the "PI" method.

What, fundamentally, is the difference between deciding in advance what rate of return is acceptable, and then calculating to see if such a rate is achieved (as in the annual cost method as commonly used), and in calculating beforehand the actual rate of return and then deciding whether or not it is acceptable, as in the "PI" or "Discounted Cash Flow" method? In both cases the same basic decision, as to how high a return must be in order to be attractive, must be made.

If it is felt that there is merit in calculating the rate of return before making this decision, the average annual cost method can be used, solving for the rate of return. This is not generally done because it increases the arithmetic required, without eliminating the necessity for deciding upon a minimum acceptable return.

That it is possible to solve an average annual cost problem for the interest rate should not be surprising, since any present worth solution of a compound interest problem can very easily be transformed into an annual cost solution.

Mr. Reul also goes on to complain that the average annual cost, as well as the capitalized cost, method, yields solutions that are "qualitative rather than quantitative". Since both such solutions are expressed in numbers (numbers that can be directly related to those offered by a present worth approach) it is difficult to understand this statement. Actually, these three approaches are different window dressings for the same thing. This allegation is similar to saying that the solution for x in

$$\frac{K}{x} = A$$

is quantitative, but that the solution for x in

$$Ax = K$$

is qualitative.

In speaking of the MAPI formula, Mr. Reul is quite harsh. He says that "This approach is an interesting intellectual flight of the imagination. Many of the assumptions necessary to make its application practical are unrealistic . ." In another place he speaks of "the error of using . . [an] inferiority gradient as [a]

factor in the evaluation of a specific investment proposal" and goes on to say that "The point is that the evaluation of an investment opportunity is an attempt to measure the future economic significance of one decision made now. We are not concerned with possible future alternatives."

Surely the fact that a new and much improved machine may be in the offing will influence, quite properly, a decision on whether or not to purchase today's machine. Would not your decision to buy a black and white television set be influenced by the knowledge that a color set, at the same price, might shortly become available? Or again, suppose you are considering the purchase of a special purpose machine for making vacuum tubes. Would the fear of technological obsolescence be irrelevant?

It is true, as Mr. Reul states, that the MAPI formula assumes "uniform regular changes . . . that are not realistic" in the estimation of the inferiority gradient. But so does the "PI" method. Whether intentional or not, the mathematics of this method requires the assumptions that there will be no technological improvement. That is, in Terborgh's terms, the inferiority gradient will be zero. These assumptions are just as arbitrary, and just as rigid, as the MAPI assumptions that technological change will occur at a positive uniform annual rate. The question that faces the analyst is which approach is based on the more realistic assumptions. (Surely it is too much to ask that all assumptions on which any mathematical model is based be completely realistic.)

In different cases, this question will be answered differently. Enthusiasm for any one particular approach should not blind

us to the fact that, in specific cases, others might be more appropriate.

The Harvard Business Review article also states that the MAPI method "requires the assumptions of a constant interest rate on which all decisions must be based even though the acceptable rate will vary with the risk involved". This just is not true. The analyst can use any rate that seems appropriate. And furthermore, as Orensteen has shown[1], the concept of an inferiority gradient is one way of allowing for risk in a replacement calculation.

In summary, the "PI", or "Discounted Cash Flow" approach is <u>one</u> satisfactory way of approaching <u>some</u> capital expenditure problems. Other methods, some based on differing assumptions, also have their place. Many of the dissimilarities among the various approaches are more apparent than real. What is needed, therefore, is a more objective analysis of the strengths and weaknesses of all approaches, rather than a panegyric for one, coupled with the damnation of all others.

[1] ORENSTEEN, ROGER BERTRAM: "Topics on the MAPI Formula", The Journal of Industrial Engineering, Volume VII, Number 6, pp. 283-294, November-December 1956.

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A SAMPLING OF THE RECENT LITERATURE OF ENGINEERING ECONOMICS IN THE TOOL PRODUCTION ENGINEERING AREA

By

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The following are brief reviews of recent articles appearing in The Tool Engineer magazine, on subjects in engineering economics. The Tool Engineer was selected because it is as comprehensive as any publication in the field and more inclined than most to give space to mathematical treatments. Most of the material intended for practicing engineers and technicians in industry must be of the recipe type to gain much attention, but it is not all of this kind, as is evidenced specifically in the following abstracts.

F. J. LANGIER: "Determining Economic Lot Size", The Tool Engineer, Volume 40, Number 2, pp. 116-18, February 1958.

The author gives a formula for economic lot size, without derivation, and a table of values to simplify use of the formula. A comparison is made to show the similarity of Langier's formula with one developed by E. C. Varnum ("How to Pick Economic Lot Size", The Tool Engineer, pp. 85-8, November 1956).

B. A. MARGO: "New Approach to Replacement Studies", The Tool Engineer, Volume 40, Number 1, pp. 73-8, January 1958.

A new word is coined for replacement studies: "replaciation", which "has to do with the anticipated cost of replacements which should normally be earned by including appropriate amounts in the

selling price of the products". This is in contrast to depreciation, which still would have to be considered for conventional and tax accounting purposes. How the new concept may be handled seems to be summarized in the following paragraph from the article:

"This implies that two records should be kept for depreciation; one in nominal dollars, the other in amounts adjusted according to the varying purchasing power of money. A similar system has been in use in Sweden for many years. It required correction some time ago because it gave too strong an incentive for investment in new machinery, etc. It had an inflationary effect. However, it is not too difficult to design a system which would prevent this possibility."

A discussion of replacement charges in replacement studies suggests that the future costs of both existing and proposed machines take into account the replacement cost of each. Also, it is suggested that possible changes in labor costs and possible savings in future taxes be taken into account. Just how these items would be calculated is not stated in detail, and no example is given. In essence, this article seems to point to some changes in accounting and economic analysis procedures, to correct for variations in the purchasing power of money; and it presents persuasive reasons for their need without much specification of detail.

FRANK J. GALLAGHER: "Calculating Economic Cutting Speeds", The Tool Engineer, Volume 40, Number 1, pp. 98-102, January 1958.

Nothing new pertaining to the theory of economical tool life is presented here. An expression is set up for the time equivalent

to the cost of a metal cutting operation, in terms of the time needed for handling, machining, and tool replacement. The expression is differentiated with respect to the cutting speed (or tool life), and the derivative equated to zero to derive an expression for the minimum cost cutting speed (or tool life). This same derivation has been given in many books and papers heretofore. The contribution of this paper is a nomograph and system to simplify calculations in using the formula.

A. N. COLE: "Realistic Depreciation Policies are Vital to Survival", The Tool Engineer, Volume XXXIX, Number 3, pp. 81-6, September 1957.

The case for accelerated depreciation allowances, to compensate for inflation in equipment costs and ease tax burdens, has been made in many guises. This seems to be one of the better reasoned ones, tinged with the viewpoint of the accountant rather than the engineer. Inventory write-off by the last-in-first-out method (LIFO), as opposed to first-in-first-out method (FIFO), is explained, and a plea is made to work towards acceptance of some similar plan for capital assets. Some logical approximations to such a method are suggested, but the actual plan is not spelled out because the author recognizes that the mechanics would have to be evolved. The beneficial results of such a plan, upon the earning, tax, and cash positions of a hypothetical concern, in a period of rising prices, is shown by an example. What the effect and what the attitude of business men would be in a period of falling prices is not mentioned.

RALPH H. ESHELMAN: "Make or Buy? - Find the Answer with OR", The Tool Engineer, Volume XXXIX, Number 1, pp. 73-8, July 1957.

The observation has been made that so-called operations research is often merely doing what engineers have done for many years in industry. The author really did not have to resort to the questionable use of the name "OR" in order to glamorize a well prepared report on sound applications of everyday engineering economics in a manufacturing plant.

In effect, this article reports how engineers in one organization systematized the making of analyses of economic alternatives but at the same time assured a high degree of reliability of the results. One of the first things found was that the plant's cost accounting system gave little information of value for the needs of the study. There follows a fairly detailed description of what was needed and done to obtain significant figures on pertinent costs. The report says that after a system was set up to extract the needed cost figures from the data coming in from the shop, "a procedure was developed to continue gathering of these data, so that they would improve in accuracy as time went on". No mention is made of the use of any statistical techniques to test or indicate the accuracy of the cost figures (which one would expect in a true OR program).

The cost figures applicable to each part made are assembled on a form, an example of which is given. It provides the usual means of assembling and listing the costs relevant to each alternative, make or buy. It is explained that the cost figures alone do not determine the final decision. A committee, comprised of

representatives from the staff functions concerned, weighs the stated costs along with many other factors, in arriving at a decision. This article is noteworthy in that it includes a good account of what some of the more important of these considerations are and how they influence decisions, illustrated by specific examples.

MARTIN H. GREENBERGER: "Machinery Replacement Decisions - Developing and Using Formulas", The Tool Engineer, Volume XXXIX, Number 1, pp. 95-100, July 1957.

The author presents an easy to follow derivation of the basic MAPI formulas, for both salvage and nonsalvage residues. The discussion concludes with consideration of straight line depreciation, exponental decay, and the declining balance method in connection with the salvage value formula.

A derivation of the same formulas (but in a different manner) was previously given by Edward C. Varnum in "Machinery Replacement - How Costs are Determined", The Tool Engineer, p. 70, June 1953. A publication like The Tool Engineer must serve an audience with large differences in interests and abilities. As a result, much of its contents must be pragmatic and ephemeral. It is indicative of the good editorial policy and management of this magazine that basic and significant treatises such as these are made available to competent tool engineers who normally would not have ready access to economic or mathematical journals.

contd.

S. E. SCHARFF: "Determining Total Costs of Machine Replacement", The Tool Engineer, Volume XXXVIII, Number 6, pp. 73-7, June 1957.

The author is associated with a leading firm of accountants and undertakes to enlighten the tool engineer about some of the considerations in machine replacement analysis from the accountant's viewpoint. A single and simple example carried throughout the discussion serves to illustrate the points clearly. The payoff period is selected as the criterion for judging the desirability of replacement, and is calculated on the basis of the savings in direct costs and overhead. It is assumed that that is as far as a hypothetical tool engineer might go. The discussion then points out specific factors which might and do affect the savings, such as interest, certain indirect costs, savings from quantity purchases of materials, materials handling, etc. This is followed by a brief explanation of common methods of depreciation, to show how the amount of estimated savings above depreciation may be considered an addition to profits.

Perusal of this article brings to mind a criticism that has been made of political cartoons, that they do not express a needed "Yes, but". In this case, the author admits that the treatment is somewhat simplified to serve the purpose of illustrating the danger of using only direct labor costs. The shortcoming here lies in the inability, within the space available, to show the many additional pitfalls in the broad area scanned.

RAYMOND G. RUSHING: "Will Automation Pay?", The Tool Engineer, Volume XXXVIII, Number 6, pp. 108-15, June 1957.

A basis for justifying or rejecting automation in any particular

case should obviously be a major concern of engineering economics today. This paper provides an apparently universal approach to the solution of that problem. In that respect it is a noteworthy contribution to the applications of engineering economy in tool engineering.

A procedure for determining whether and how much to automate a process is outlined as follows:

"1. Study and become thoroughly familiar with the operations in question.
"2. Assume the entire operation is completely automated and/or partially automated regardless of how impossible such might appear.
"3. Analyze the time balance required between operations based on existing equipment incorporating any known applicable automatic devices.
"4. Determine how many machines are required for continuous flow production.
"5. Determine the actual machine time required with existing equipment incorporating any known applicable automatic devices.
"6. Determine the total time presently allowed an operator to complete required production.
"7. Determine any savings in manufacturing costs that might be realized through automation.
"8. Determine the cost of incorporating mechanization of each operation to obtain automation.
"9. Compare the savings on manufacturing costs with the cost of incorporating automation.
"10. Evaluate the automated set-up and make a decision."

The remainder of the article is devoted to a detailed exposition of the use of this procedure in a specific case. Of particular interest is the author's explanation of how algebraic relationships may be developed to analyze the time balance required between operations to carry out step 3. Curves of these relationships, with values inserted for the particular case, lead to the solution of step 4, the determination of the machines required.

The example is appropriately arranged to show that the form of analysis proposed not only provides a basis for deciding

whether to automate or not, but also a basis for evaluating the extent to which automation is justified.

In the concluding paragraph of the paper, the author gives this deserved commendation to the proposed procedure:

"Advantages of this kind of approach to mechanization and automation are that the problem is simplified by breaking it down into easily understood elements; a reasonably accurate picture is secured of pros and cons of any proposed improvement; and finally, specific cost data is obtained to present to management. When a completely automatic installation lacks economic justification, as in the example cited, this kind of study will uncover alternatives that may provide gains even greater than anticipated."

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A SIMPLIFIED FORMULA FOR BREAK-EVEN POINTS

By

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The break-even approach as developed by Professor Rautenstrauch relies upon the graphical approach.[1] It is possible, however, to derive a formula for break-even points which is simple and easy to apply, using the same kinds of assumptions as Professor Rautenstrauch.

The basic formula applies to the single commodity case, where the problem is to find the break-even percentage of capacity, and is as follows:

$$(1) \quad \beta = \alpha\left(1 - \frac{\pi_o}{m_o}\right) \qquad \text{(Footnote 2)}$$

where β = break-even point as per cent of capacity
α = current rate of operations as per cent of capacity
π_o = current rate of profits before taxes as per cent of sales
m_o = gross margin, i.e., sales minus variable costs, as per cent of sales

To illustrate formula (1) with a numerical illustration: Suppose a plant has a capacity of 500,000 units, a fixed cost of $100,000, a selling price of $1, and an average variable cost of 50 cents. Suppose further that the plant is working at capacity; then $\alpha = 1$, $\pi_o = 30\%$, and $m_o = 50\%$. Hence, $\beta = \left(1 - \frac{30\%}{50\%}\right) = 40\%$. Suppose, however, that the plant is working at 50% of capacity.

[1] WALTER RAUTENSTRAUCH and RAYMOND VILLERS: The Economics of Industrial Management, Chapter IV. Funk and Wagnall, New York, 1949. 451 pp. $7.50 (Text edition $6).

[2] See Appendix for derivation of formula.

In this case, $\alpha = \frac{1}{2}$, $\pi_o = 10\%$, and $m_o = 50\%$ as before. Hence, $\beta = \frac{1}{2}\left(1 - \frac{10\%}{50\%}\right) = 40\%$, as before.

The nature of the assumptions involved in the derivation of this formula becomes clear from the example. The formula assumes that price and average variable costs are constant. These are the assumptions most commonly made in derivation of break-even points.[1] Because this formula assumes, in effect, a constant product mix, how good an approximation it is will therefore vary with the circumstance. However, the formula is flexible enough so that adjustments can be made in the basic parameters to allow for changes in the fundamental cost-price-volume relationships. On the other hand, the simplicity of the formula is such that it eliminates the time consuming process of deriving statistical sales and cost functions.

The second case in which the formula can be applied is in the derivation of break-even levels of output. This is another aspect of the single commodity case. The formula appropriate to that case is:

$$(2) \quad O_B = O_N \left(1 - \frac{\pi_o}{m_o}\right)$$

where O_B = break-even level of output

O_N = current level of output

Finally, the most common case is the multi-product case where the break-even level of dollar sales is sought. The formula appropriate to this case is:

[1] CLAYTON W. ANDERSON, "Disclosure of Assumptions - Key to Better Break-Even Analysis", NAA Bulletin, Section 1, p. 25, December 1957. National Association of Accountants, 505 Park Avenue, New York 22, N.Y. $10 yearly.

$$(3) \quad S_B = S_N \left(1 - \frac{\pi_o}{m_o} \right)$$

where S_B = break-even level of sales

S_N = current level of sales

It is interesting to note that the relation between the profit rate and the other factors can be easily derived:

$$(4) \quad \pi_o = \left(1 - \frac{\beta}{\alpha} \right) m_o$$

Thus, if the break-even point and the gross margin (m_o) are given, the effect of a change in the sales volume on the profit rate (π_o) can be determined. Furthermore, it should be noted from (4) that π_o varies inversely with the break-even point. It thus follows, as one would expect, that the lower the break-even point, the shorter the payout period for a new investment.

The formulas developed here may possibly have particular use in making comparative analyses of the break-even points of different companies. While it is true that published income statements do not give sufficient detail to make an exact determination of break-even points, since plant overhead is not shown separately, it is possible to make rough approximations rather quickly through this formula.

APPENDIX

Derivation of Formula

At the break-even point, sales equal total costs, i.e.,

$$S = V + F$$

where S = sales
V = total variable costs
F = fixed costs

Hence $S - V = F$

i.e., $O_B (p - V\alpha) = F$

where O_B = break-even level of output

p = price or average sales revenue

$V\alpha$ = average variable cost

At any level of output

$$S = V + F + G$$

where G = total profits before taxes

i.e., $S - V - G = F$

$$O_N (p - V\alpha - \pi) = O_B (p - V\alpha)$$

where O_N = current level of output

π = average dollar profit before taxes per unit

Dividing through by $(p - V\alpha)$

$$O_B = O_N \left(1 - \frac{\pi}{p - V\alpha} \right)$$

Dividing through by the capacity level of output:

$$\beta = \alpha \left(1 - \frac{\pi_o}{m_o} \right)$$

where $\pi_o = \frac{\pi}{p}$, $m_o = \frac{p - V\alpha}{p}$

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THE ENGINEERING ECONOMIST

INDEX TO VOLUME 3: SUBJECT MATTER

	Issue	Page
CAPITAL BUDGETING		
Capital Budgeting and the Problem of Reinvesting Cash Proceeds - Reviewed by G. C. McKeague	3	51
False and Multiple Solutions by the Discounted Cash Flow Method for Determining Interest Rate of Return - James B. Weaver	4	1
Profitability Index for Investments - Reviewed by Ralph O. Swalm	4	40
CAPITAL INVESTMENT		
A Critique of Allowances for Uncertainty in Estimating the Productivity of Capital on Individual Investments - Fred W. Cleveland, Jr.	3	1
Judging Profitability by Venture Worth or Incremental Worth - Milton B. Rogers	2	47
DECISION MAKING		
Use of the Monte Carlo Technique for the Engineering Economist - S. B. Eubank and James M. White	2	1
DEPRECIATION		
Rapid Amortization of Emergency Facilities - Hearings Before the Committee on Finance, United States Senate - David A. Thompson	1	39
Depreciation Regulations - Internal Revenue Treasury Decision 6182 - Table of Contents	3	59
ECONOMICS		
Economic Models - Reviewed by Paul W. McGann	2	54
INVENTORY ECONOMICS		
Scientific Inventory Management Simplified - The Economic Order Quantity - Reviewed by W. C. Hugli, Jr.	2	51

	Issue	Page
ECONOMIC COMPARISONS		
Appraising the Economics of Electronic Computers - Reviewed by Irving M. Jasphy	3	55
A Sampling of the Recent Literature of Engineering Economics in the Tool Production Engineering Area - Lawrence E. Doyle	4	47
Engineering Economy, Second Edition - Reviewed by Henry P. Goode	1	52
The Economic Design of $\bar{X}$ Charts Used to Maintain Current Control of a Process: Model I - Reviewed by L. G. Mitten	3	43
REPLACEMENT		
Equipment Replacement and Depreciation Policy - Gerald J. Matchett	1	34
Estimating and Post-Auditing Direct Labor Savings in a Job-Shop Operation - Raymond R. Mayer		
Part I	2	18
Part II	3	15
TAXATION		
Handling of Income Taxes in Engineering Economy Studies From the Public Utility Viewpoint - William J. Murphy	1	13
Why Must Taxes Be Considered in Profitability Studies - James B. Weaver	1	1
COST ANALYSIS		
Cost Concepts for Business Management - James L. Lundy	3	47
BREAK-EVEN ANALYSIS		
A Simplified Formula for Break-Even Analysis - Benjamin Caplan	4	55
Modernizing the Break-Even Analysis - Ernest H. Weinwurm	4	32

* * *

THE ENGINEERING ECONOMIST

INDEX TO VOLUME 3 : AUTHORS

NAME	Issue	Page
Caplan, Benjamin	4	55
Cleveland, Fred W., Jr.	3	1
Doyle, Lawrence E.	4	47
Eubank, S. B. and James M. White	2	1
Goode, Henry P.	1	52
Hugli, W. C., Jr.	2	51
Jasphy, Irving M.	3	55
Lundy, James L.	3	47
McGann, Paul W.	2	54
McKeague, G. C.	3	51
Matchett, Gerald J.	1	34
Mayer, Raymond R.	2 3	18 15
Mitten, L. G.	3	43
Murphy, William J.	1	13
Rogers, Milton B.	2	47
Swalm, Ralph O.	4	40
Thompson, David A.	1	39
Weaver, James B.	1 4	1 1
Weinwurm, Ernest H.	4	32
White, James M. and S. B. Eubank	2	1

* * *